## Conductance modulation in spin field-efect transistors under finite bias voltages

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The conductance modulations in spin field-effect transistors under finite bias voltages were studied. It was shown that when a finite bias voltage is applied between two terminals of a spin field-effect transistor, the spin precession states of injected spin-polarized electrons in the semiconductor channel of the device will depend not only the gate-voltage controlled Rashba spin-orbit coupling but also depend on the bias voltage and, hence, the conductance modulation in the device due to Rashba spin-orbit coupling may also depend sensitively on the bias voltage.

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## I. INTRODUCTION

In the recent years spin-polarized transport in semiconductor microstructures has attracted much attention because of its important relevance to the emerging field of spintronics, a new branch of electronics where the electron's spin (in addition to its charge) is the active element for information storage and processing.<sup>1</sup> An issue of fundamental importance in the emerging field of spintronics is the generation and control of high spinpolarized currents in semiconductors. 1,2,3,4,5 Recently high efficient injection of spin-polarized currents from magnetic to non-magnetic semiconductors have been achieved at low temperatures; however, efficient injection of spin-polarized currents from ferromagnetic (F) metals into semiconductors (S) has not yet been realized experimentally. But for room temperature spintronic devices, ferromagnetic metal sources are indispensable tools. Detailed theoretical investigations have revealed that the main obstacle for spin injection from a F metal source into a semiconductor originates from the large mismatch between the conductivities of metals and semiconductors.<sup>7,8</sup> It can be show that in a usual F metal/semiconductor heterojunction, the spin injection coefficient is proportional to  $\sigma_S/\sigma_F$ , where  $\sigma_S$  and  $\sigma_F$  are the conductivity of the semiconductor and the F metal, respectively. Since  $\sigma_S \ll \sigma_F$ , the efficiency of spin injection in usual F metal/semiconductor heterojunctions is very small. At first glance, this problem seems insurmountable, but very recent theoretical investigations show that this obstacle may be overcome through the use of suitable potential barriers<sup>8,9,10</sup> or through appropriate epitaxial interfaces that obey certain selection rules and band structure symmetry properties<sup>11,12</sup>, and encouraging experimental results have also been obtained following the theoretical predictions <sup>13,14,15</sup>. These results suggest that devices made of combinations of F metals and semiconductors may be truly promising for applications in spintronics. Among the most prominent device proposals that involve combinations of F metals and semiconductors is the spin field-effect transistor (spin FET) $^4$ . In a spin FET, two ferromagnetic metallic electrodes are coupled via a ballistic semiconductor channel. The current modulation in the structure arises from spin precession of injected spin-polarized electrons in the semiconductor channel due to Rashba spin-orbit coupling, while two ferromagnetic metallic electrodes are used to preferentially inject and detect the spin-polarized currents. It has long been established both theoretically <sup>16,17</sup> and experimentally 18,19 that, arising from the structural inversion asymmetry, there is a spin-orbit interaction in two dimensional electron gases (2DEGs) on narrow-gap semiconductor ( such as InAs ) surfaces. This underlying spin-orbit interaction was known as Rashba spinorbit coupling in the literatures. An important feature of Rashba spin-orbit coupling is that its strength can by tuned by an external gate voltage, which alters the buildin structural inversion asymmetry. Due to this fact, spin precession of injected spin-polarized electrons in the S channel of a spin FET can be tuned by applying an external gate voltage, and concomitantly, the current flowing through the device can be also modulated. This mechanism was first proposed in a seminal work by Datta and Das<sup>4</sup> and recently, some important factors that will affect the behaviors of a spin FET were investigated in more details and with more realistic assumptions. 20,21,22,23,24,25 In the present paper, we discuss the conductance modulations in spin FETs under finite bias voltages. Previous theoretical investigations have been focussed on the zero-bias conductance modulations in spin FETs, but in practical applications a finite bias voltage need to be applied between both terminals of a spin FET, and the conductance-bias voltage characteristics of a device are usually very important for practical applications of the device. From theoretical viewpoints, when a finite bias voltage is applied between two terminals of a spin FET, a longitudinal electric field will be established in the semiconductor channel of the device, and as was well known, in spin-orbit coupled systems, external electric field may play a more subtle role on electron's transport than in traditional electronic devices since in spin-orbit coupled systems the effect of electric field may be sensitively spindependent. (Examples of unusual effect of electric field on electron's charge and spin transport in spin-orbit coupled systems can be seen from Refs. 26,27,28. ) In the present paper we discuss the influence of finite bias voltages on the conductance modulations in spin FETs due to Rashba spin-orbit coupling. We will show that if a finite bias voltage is applied between two terminals of a spin

FET, the conductance modulation in the structure due to Rashba spin-orbit coupling may depend sensitively on the bias voltage, and in order to describe correctly the spin precession state of injected spin-polarized electrons in the semiconductor channel, the interplay between the Rashba spin-orbit coupling ( which can be tuned via the gate voltage ) and the longitudinal electric field induced by the application of a finite bias voltage should be described in a unified way.

## II. MODEL AND FORMULATION

For simplicity, in this paper we will restrict ourselves to a one-dimensional (1D) model. In one-dimensional systems the quantum interference effect due to Rashba spinorbit coupling will be maximum since the phase shifts of electrons are independent of their paths, so the idealized 1D model will give an upper limit for the achievable spintransistor effect. In higher dimensions, the phase shifts of electrons will depend on their paths and, hence, the spin-transistor effect will become weaker than what is predicted in a 1D model system. This was illustrated in Ref.? Though in the present paper we restrict our discussion to a 1D model system, the formulas given below are easy to be extended to systems with higher dimensions. This will be discussed elsewhere. In the one-band effective-mass approximation, the 1D model system can be described by the following Hamiltonian:

$$\hat{H} = \frac{1}{2}\hat{p}_x \frac{1}{m(x)}\hat{p}_x + \frac{1}{2\hbar}\hat{\sigma}_z[\hat{p}_x\alpha(x) + \alpha(x)\hat{p}_x]$$

$$+ \frac{1}{2}\Delta\hat{\boldsymbol{\sigma}}\cdot[\vec{m}_L\theta(-x) + \vec{m}_R\theta(x-L)] + \delta E_c\theta(x)\theta(L-x)$$

$$+ \hat{U}[\delta(x) + \delta(x-L)] + V(x).$$
(1)

Here  $\theta(x)$  is the usual step function and  $\delta(x)$  the usual  $\delta$ function,  $\hat{p}_x$  is the momentum operator,  $\hat{\boldsymbol{\sigma}}$  are the Pauli matrices,  $m(x) = m_f + (m_s - m_f) \theta(x)\theta(L - x)$  is the effective mass of electron, with  $m_f$  denoting the effective mass of electron in the ferromagnetic contacts and  $m_s$  the effective mass of electron in the semiconductor channel, and the F/S interfaces are assumed to be located at x = 0 and x = L. The second term in Eq.(1) describes the Rashba spin-orbit coupling<sup>21,22,23,24</sup>, where  $\alpha(x)$  is defined by  $\alpha(x) \equiv \alpha_R \theta(x) \theta(L-x)$ , and  $\alpha_R$  is the Rashba spin-orbit coupling constant in the S region. Since the Hamiltonian  $\hat{H}$  has to be an Hermitian operator, in Eq.(1) we have used the symmetrized version of Rashba spin-orbit interaction. The third term in Eq.(1) describes the exchange interaction in the ferromagnetic contacts, with  $\Delta$  denoting the spin-splitting energy and the unit vector  $\vec{m}_L$  (  $\vec{m}_R$  ) denoting the direction of the magnetization in the left (right) contact. It will be assumed that  $\vec{m}_L$  is in the +x direction and  $\vec{m}_R$  will be in either +x direction (parallel configuration) or -xdirection (antiparallel configuration). The fourth and fifth terms in Eq.(1) model the conduction band mismatch and the interfacial scattering between the F and

S regions, respectively, with  $\delta E_c$  denoting the band mismatch and U the interfacial scattering potential. In the presence of both spin-conserving and spin-flip interfacial scattering,  $\hat{U}$  will be a 2 × 2 matrix with the diagonal elements  $(U^{\uparrow\uparrow}, U^{\downarrow\downarrow})$  representing the spin-dependent strength of spin-conserving interfacial scattering and the off-diagonal elements  $(U^{\uparrow\downarrow}, U^{\downarrow\uparrow})$  the strength of spin-flip interfacial scattering. For simplicity, we will assume that  $U^{\uparrow\uparrow} = U^{\downarrow\downarrow}$  and  $U^{\uparrow\downarrow} = U^{\downarrow\uparrow}$ . (For magnetically active interface, its is possible that  $U^{\uparrow\uparrow} \neq U^{\downarrow\downarrow}$  and  $U^{\uparrow\downarrow} \neq U^{\downarrow\uparrow}$ . Finally, the last term in Eq.(1) denotes the longitudinal electric potential due to the application of a finite bias voltage, and the longitudinal electric potential is given by  $V(x) = -eV_0\theta(x-L) - eV_0(x/L)\theta(x)\theta(L-x)$ , where  $V_0$ is the magnitude of the applied bias voltage. Due to the application of the bias voltage  $V_0$ , a longitudinal electric field  $F \equiv V_0/L$  will be established in the semiconductor channel of the structure and the Fermi energy  $\mu_R$  in the right contact will be lowered by  $eV_0$  with respect to the Fermi energy  $\mu_L$  in the left contact.

To obtain the spin conductance of the device described by the Hamiltonian (1), we start by considering the scattering problem related to the interfaces between the F and S regions. In order to solve the scattering problem, one need to find first the eigenstates in each region. In the ferromagnetic contacts ( x < 0 and x > L ), one obtains from the Hamiltonian (1) the eigenstates with energy E,

$$\Psi_{F,\sigma,L}^{(\pm)} = \phi_{F,\sigma,L}^{(\pm)}(x)|\sigma\rangle, \, \phi_{F,\sigma,L}^{(\pm)}(x)$$

$$= \sqrt{\frac{m_f}{\hbar k_{\sigma,L}}} e^{\pm ik_{\sigma,L}x}, \, (x < 0), \qquad (2)$$

$$\Psi_{F,\gamma,R}^{(\pm)} = \phi_{F,\gamma,R}^{(\pm)}(x)|\gamma\rangle, \, \phi_{F,\gamma,R}^{(\pm)}(x)$$

$$= \sqrt{\frac{m_f}{\hbar k_{\gamma,R}}} e^{\pm ik_{\gamma,R}x}, \, (x > L), \qquad (3)$$

where  $|\sigma\rangle$  ( $\sigma=\pm$ ) and  $|\gamma\rangle$  ( $\gamma=\pm$ ) are the spinor eigenstates in the left and right ferromagnetic contacts, respectively, which are defined by

$$\{|+\rangle_L, |-\rangle_L\} = \frac{1}{\sqrt{2}} \begin{pmatrix} \pm 1\\ 1 \end{pmatrix},$$
 (4a)

$$\{|+\rangle_R, |-\rangle_R\} = \lambda \frac{1}{\sqrt{2}} \begin{pmatrix} \pm 1\\ 1 \end{pmatrix},$$
 (4b)

where  $\lambda=+1$  if the two ferromagnetic electrodes are in parallel configuration and  $\lambda=-1$  if the two electrodes are in antiparallel configuration. The wave number  $k_{\sigma,L}$  ( $k_{\gamma,R}$ ) will be given by  $k_{\pm,L(R)}=\sqrt{\frac{2m_f}{\hbar^2}(E\mp\Delta)}$ . The eigenfunctions in the S region cannot be written down directly from the Hamiltonian (1) due to the presence of the last term in Eq.(1). To find the eigenstates in the S region, we first note that in the S region the Hamiltonian (1) is spin-diagonal and the eigenstates have the form  $\Psi_{S,\beta}(x)=\phi_{S,\beta}(x)|\bar{\beta}\rangle$  and  $\Psi_{S,\bar{\beta}}(x)=\phi_{S,\bar{\beta}}(x)|\bar{\beta}\rangle$ , where  $|\beta\rangle=(1,0)$  and  $|\bar{\beta}\rangle=(0,1)$  are the spinor eigenstates in

the S region. The Schrödinger equation in the S region will reduce to

$$-\frac{\hbar^{2}}{2m_{s}}\frac{\partial^{2}}{\partial x^{2}}\phi_{S,\beta}(x) - i\alpha_{R}\frac{\partial}{\partial x}\phi_{S,\beta}(x) - \frac{eV_{0}x}{L}\phi_{S,\beta}(x)$$

$$= E\phi_{S,\beta}(x), \qquad (5)$$

$$-\frac{\hbar^{2}}{2m_{s}}\frac{\partial^{2}}{\partial x^{2}}\phi_{S,\bar{\beta}}(x) + i\alpha_{R}\frac{\partial}{\partial x}\phi_{S,\bar{\beta}}(x) - \frac{eV_{0}x}{L}\phi_{S,\bar{\beta}}(x)$$

$$= E\phi_{S,\bar{\beta}}(x). \qquad (6)$$

After making a transformation  $\phi_{S,\beta}(x) \to w_{\beta}(x) = \phi_{S,\beta}(x)e^{i\alpha_R mx/\hbar^2}$  and  $\phi_{S,\bar{\beta}}(x) \to w_{\bar{\beta}}(x) = \phi_{S,\bar{\beta}}(x)e^{-i\alpha_R mx/\hbar^2}$ , it can be shown that both  $w_{\beta}(x)$  and  $w_{\bar{\beta}}(x)$  will satisfy the following equation

$$\frac{\partial^2}{\partial x^2}w(x) + \frac{2eV_0m_s}{L\hbar^2}(x+\epsilon_0)w(x) = 0, \tag{7}$$

where  $\epsilon_0$  is defined by

$$\epsilon_0 = \frac{EL}{eV_0} + \frac{\alpha_R^2 m_s L}{2eV_0 \hbar^2}.$$
 (8)

Eq.(7) can solved with the help of the Airy functions and the two linearly independent solutions can be given by  $Ai[-(2eV_0m_s/L\hbar^2)^{1/3}(x+\epsilon_0)]$  and  $Bi[-(2eV_0m_s/L\hbar^2)^{1/3}(x+\epsilon_0)]$ . Here Ai[z] and Bi[z] are the usual Airy functions<sup>29</sup>. Then one can see that there are four eigenstates in the S region, and the corresponding eigenfunctions  $\Psi_{S,\beta}^{(i)}(x)$  and  $\Psi_{S,\bar{\beta}}^{(i)}(x)$  (i=1,2) will be given by

$$\begin{array}{lll} \Psi_{S,\beta}^{(i)}(x) & = & \phi_{S,\beta}^{(i)}(x)|\beta\rangle, \ \phi_{S,\beta}^{(i)}(x) = e^{-i\alpha_R mx/\hbar^2} w^{(i)}(x)\mbox{,} \\ \Psi_{S,\bar{\beta}}^{(i)}(x) & = & \phi_{S,\bar{\beta}}^{(i)}(x)|\bar{\beta}\rangle, \phi_{S,\bar{\beta}}^{(i)}(x) = e^{i\alpha_R mx/\hbar^2} w^{(i)}(x)\mbox{,} \\ \end{array}$$

where 
$$w^{(1)}(x) \equiv Ai[-(2eV_0m_s/L\hbar^2)^{1/3}(x+\epsilon_0)]$$
 and  $w^{(2)}(x) \equiv Bi[-(2eV_0m_s/L\hbar^2)^{1/3}(x+\epsilon_0)].$ 

Now we consider the scattering state of an electron with energy E and spin  $\sigma$  incoming from the ferromagnetic lead ( x < 0 ). The total wave function including the reflected and transmitted waves can be written as:

$$\Psi_{F}(x) = \phi_{F,\sigma,L}^{(+)}(x)|\sigma\rangle + r_{\sigma\sigma}\phi_{F,\sigma,L}^{(-)}(x)|\sigma\rangle 
+ r_{\sigma\bar{\sigma}}\phi_{F,\bar{\sigma},L}^{(-)}(x)|\bar{\sigma}\rangle, \quad x < 0, \qquad (11)$$

$$\Psi_{S}(x) = \sum_{i=1,2} c_{i,\beta}\phi_{S,\beta}^{(i)}(x)|\beta\rangle 
+ \sum_{i=1,2} c_{i,\bar{\beta}}\phi_{S,\bar{\beta}}^{(i)}(x)|\bar{\beta}\rangle, \quad 0 < x < L, \qquad (12)$$

$$\Psi_F(x) = t_{\sigma\gamma}\phi_{F,\gamma,R}^{(+)}(x)|\gamma\rangle + t_{\sigma\bar{\gamma}}\phi_{F,\bar{\gamma},R}^{(+)}(x)|\bar{\gamma}\rangle, \quad x > L,$$
(13)

where  $r_{\sigma\sigma}$ ,  $r_{\sigma\bar{\sigma}}$ ,  $t_{\sigma\gamma}$ ,  $t_{\sigma\bar{\gamma}}$ ,  $c_{i,\beta}$ , and  $c_{i,\bar{\beta}}$  ( i=1,2 ) are coefficients that need to be determined by the boundary

conditions. The matching conditions at the interfaces between the F and S regions can be obtained by integrating  $\hat{H}\Psi=E\Psi$  from  $x=-\varepsilon$  to  $x=+\varepsilon$  and from  $x=L-\varepsilon$  to  $x=L+\varepsilon$  in the limit  $\varepsilon\to 0$ . This yields

$$\Psi_F(x)|_{x=0^-} = \Psi_S(x)|_{x=0^+}, \tag{14}$$

$$\Psi_S(x)|_{x=L^-} = \Psi_F(x)|_{x=L^+},\tag{15}$$

$$\hat{v}_S \Psi_S(x)|_{x=0^+} = \hat{v}_F \Psi_F(x)|_{x=0^-} - \frac{2i}{\hbar} \hat{U} \Psi_F(x)|_{x=0^-},$$
(16)

$$\hat{v}_S \Psi_S(x)|_{x=L^-} = \hat{v}_F \Psi_F(x)|_{x=L^+} + \frac{2i}{\hbar} \hat{U} \Psi_F(x)|_{x=L^+},$$
(17)

where  $\hat{v}_F = \hat{p}_x/m_f$  and  $\hat{v}_S = \hat{p}_x/m_s + (\alpha_R/\hbar)\hat{\sigma}_z$  are the velocity operators in the F and S regions, respectively. From the matching conditions (14)-(17), the transmission coefficients  $t_{\sigma\gamma}$  can be obtained. Then in the linear response regime and in the low temperature limit, the spin conductance  $G_{\sigma}$  and the total conductance G of the device can be calculated through the Landauer formula, given by

$$G = \sum_{\sigma = +} G_{\sigma}, G_{\sigma} = \frac{e^2}{h} \sum_{\gamma = +} |t_{\sigma\gamma}(\mu)|^2,$$
 (18)

where  $\mu$  is the average of the Fermi energies  $\mu_L$  and  $\mu_B$  on the left and right electrodes, respectively<sup>30</sup>. The spin injection efficiency for the device can be defined by  $\eta = (G_+ - G_-)/(G_+ + G_+)$ . This ratio characterizes the spin polarization of the electric current flowing through the device. The conductance of the device and the spin injection coefficient will depend on the magnetization configurations in the two ferromagnetic electrodes. In the following we will denote the conductance as  $G^{(P)}$  and the spin injection coefficient as  $\eta^{(P)}$  if the magnetizations in the two electrodes are parallel and as  $G^{(A\tilde{P})}$  and  $\eta^{(AP)}$  if the magnetizations in the two electrodes are antiparallel. The change in conductance when the two ferromagnetic electrodes switch between parallel and antiparallel configurations can be measured by a magnetoconductance ratio  $\eta_M$ , defined by

$$\eta_M = \frac{G^{(P)} - G^{(AP)}}{G^{(P)} + G^{(AP)}}. (19)$$

## III. RESULTS AND DISCUSSIONS

Based on the formulas established above, in this section we will present some numerical examples by considering some actual experimental parameters. We will solve Eqs.(14)-(17) numerically by transfer matrix method. In order to obtain the transfer matrix, it may be more convenient to rewrite the wave function in the

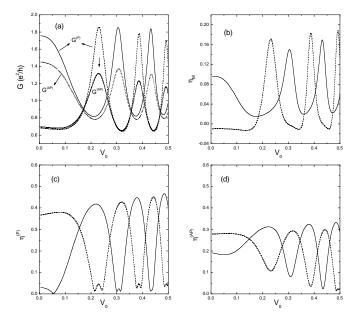


FIG. 1: The changes of (a) the conductance  $G^{(P)}$  and  $G^{(AP)}$ , (b) the magnetoconductance ratio  $\eta_M$ , and (c) the spin injection coefficient  $\eta^{(P)}$  and  $\eta^{(AP)}$ , with the variation of the bias voltage V in two cases with different Rashba spin-orbit coupling constant. (The strength of Rashba spin-orbit coupling is characterized by the Rashba wave number  $k_R \equiv m_s \alpha_R/\hbar^2$ . In Fig1.(a),  $k_R = 10^7 {\rm cm}^{-1}$  for the solid line and the dotted line,  $k_R = 5 \times 10^7 {\rm cm}^{-1}$  for the dashed and the dash-dotted line. In Fig.1(b)-(d),  $k_R = 10^7 {\rm cm}^{-1}$  for the solid line and  $k_R = 5 \times 10^7 {\rm cm}^{-1}$  for dashed line. Other parameters used were given in the text.

electrodes in a more general form as following:

$$\Psi_{F}(x) = \sum_{\sigma=\pm} [a_{\sigma}^{(+)} \phi_{F,\sigma,L}^{(+)}(x) | \sigma \rangle + a_{\sigma}^{(-)} \phi_{F,\sigma,L}^{(-)}(x) | \sigma \rangle],$$

$$x < 0, \qquad (20)$$

$$\Psi_{F}(x) = \sum_{\gamma=\pm} [b_{\gamma}^{(+)} \phi_{F,\gamma,R}^{(+)}(x) | \gamma \rangle + b_{\gamma}^{(-)} \phi_{F,\gamma,R}^{(-)}(x) | \gamma \rangle],$$

$$x > L, \qquad (21)$$

If the spin of incident electron is  $|\sigma\rangle$ , one has  $a_{\sigma}^{(+)}=1$ ,  $a_{\bar{\sigma}}^{(+)}=0$ ,  $a_{\bar{\sigma}}^{(-)}=r_{\sigma\bar{\sigma}}$ ,  $a_{\bar{\sigma}}^{(-)}=r_{\sigma\bar{\sigma}}$ ,  $b_{\gamma}^{(+)}=t_{\sigma\gamma}$ ,  $b_{\bar{\gamma}}^{(+)}=t_{\sigma\bar{\gamma}}$ .  $b_{\gamma}^{(-)}$  (  $\gamma=\pm$  ) will be set to be zero. From Eq.(12) and Eqs.(20)-(21), at the interfaces between the F and S regions,  $\Psi_F(x)$ ,  $\Psi_S(x)$ ,  $\hat{v}_F\Psi_F(x)$ , and  $\hat{v}_S\Psi_S(x)$  can be

expressed as following:

$$\begin{bmatrix} \Psi_{F}(x)|_{x=0^{-}} \\ \hat{v}_{F}\Psi_{F}(x)|_{x=0^{-}} \end{bmatrix} = \hat{S}_{1} \begin{bmatrix} a_{+}^{(+)} \\ a_{-}^{(+)} \\ a_{-}^{(-)} \end{bmatrix},$$

$$\begin{bmatrix} \Psi_{S}(x)|_{x=0^{+}} \\ \hat{v}_{S}\Psi_{S}(x)|_{x=0^{+}} \end{bmatrix} = \hat{S}_{2} \begin{bmatrix} c_{1,\beta} \\ c_{2,\beta} \\ c_{1,\bar{\beta}} \\ c_{2,\bar{\beta}} \end{bmatrix},$$

$$\begin{bmatrix} \Psi_{S}(x)|_{x=L^{-}} \\ \hat{v}_{S}\Psi_{S}(x)|_{x=L^{-}} \end{bmatrix} = \hat{S}_{3} \begin{bmatrix} c_{1,\beta} \\ c_{2,\beta} \\ c_{1,\bar{\beta}} \\ c_{2,\bar{\beta}} \end{bmatrix},$$

$$\begin{bmatrix} \Psi_{F}(x)|_{x=L^{+}} \\ \hat{v}_{F}\Psi_{F}(x)|_{x=L^{+}} \end{bmatrix} = \hat{S}_{4} \begin{bmatrix} b_{+}^{(+)} \\ b_{-}^{(-)} \\ b_{-}^{(-)} \end{bmatrix}, \qquad (22)$$

where  $\hat{S}_i$  (i = 1, 2, 3, 4) are matrices, and the matrix elements of  $\hat{S}_i$  can be written directly from Eqs.(12) and Eqs.(20)-(21). From the matching condition (14)-(17) and Eq.(22), one gets that

$$\begin{bmatrix} a_{+}^{(+)} \\ a_{-}^{(+)} \\ a_{-}^{(-)} \\ a_{-}^{(-)} \end{bmatrix} = \hat{S}_{t} \begin{bmatrix} b_{+}^{(+)} \\ b_{-}^{(+)} \\ b_{-}^{(-)} \\ b_{-}^{(-)} \end{bmatrix}, \tag{23}$$

where  $\hat{S}_t \equiv \hat{S}_1^{-1} \hat{S}_2 \hat{S}_3^{-1} \hat{S}_4$  are the transfer matrix. Taking  $b_+^{(-)} = 0$  and  $b_+^{(-)} = 0$ , then from Eq.(23) one gets that

$$\begin{bmatrix} b_{+}^{(+)} \\ b_{-}^{(+)} \end{bmatrix} = \hat{T} \begin{bmatrix} a_{+}^{(+)} \\ a_{-}^{(+)} \end{bmatrix}, \hat{T} = \begin{bmatrix} S_{t}(1,1) & S_{t}(1,2) \\ S_{t}(2,1) & S_{t}(2,2) \end{bmatrix}^{-1},$$
(24)

where  $S_t(i,j)$  are the matrix elements of the transfer matrix  $\hat{S}_t$ . Since  $a_{\sigma}^{(+)} = 1$  and  $a_{\bar{\sigma}}^{(+)} = 0$  if the spin of incident electron is  $|\sigma\rangle$ , then the transmission coefficient can be got directly from Eq.(24) as following:  $t_{++} = T(1,1)$ ,  $t_{+-} = T(2,1), t_{-+} = T(1,2), t_{--} = T(2,2), \text{ where}$ T(i,j) are the elements of the matrix  $\hat{T}$ . After the transmission coefficients are obtained, the spin conductance of the device can be got from Eq.(18). In the following we will focus on iron (Fe) as the ferromagnetic source and drain and InAs as the semiconductor channel. In the ferromagnetic electrodes the Fermi energy ( in the equilibrium state ) will be set to  $E_F = 2.469 eV$  and the exchange splitting energy be set to  $\Delta = 3.46eV$ , appropriate for Fe. The effective masses were set to  $m_f = m_e$ ( for Fe ) and  $m_s = 0.036m_e$  ( for InAs ), and the band mismatch between the F and S regions were set to  $\delta E_c = 2.0 eV$ . The length of the semiconductor channel was set to be  $1\mu m$ . The strength of Rashba spinorbit coupling will be characterized by a Rashba wave number  $k_R \equiv m_s \alpha_R / \hbar^2$ . For simplicity, we first assume that the interfacial scattering potential is zero ( $\hat{U}=0$ ). In Figs.1(a)-(b) we have plotted the changes of the total conductance  $G^{(P)}$  and  $G^{(AP)}$  and the magnetoconductance ratio  $\eta_M$  with the variation of the bias voltage V in two cases with different strengths of Rashba spin-orbit coupling, and the changes of the spin injection coefficient  $\eta^{(P)}$  and  $\eta^{(AP)}$  with the variation of the bias voltage V were also plotted in Figs.1(c)-(d), respectively. From Figs. 1(a)-(d) one can see that in a large range of the bias voltage V, the conductance and the magnetoconductance ratio and the spin injection coefficient all can be changed significantly by tuning the Rashba spinorbit coupling, i.e., the structure described the Hamiltonian (1) may exhibit significant spin-transistor effect in a large range of the bias voltage. But Figs. 1(a)-(d) show that the modulations of the conductance and the magnetoconductance ratio and the spin injection coefficient due to Rashba spin-orbit coupling may depend sensitively on the bias voltage, i.e., the changes of the conductance and the magnetoconductance ratio and the spin injection coefficient with the variation of Rashba spin-orbit coupling (which can be tuned by changing the gate voltage ) may be very different under different bias voltages. This can be seen more clearly from Figs.2(a)-(c), where we have plotted the changes of the conductance  $G^{(P)}$ and  $G^{(AP)}$  and the magnetoconductance ratio  $\eta_M$  and the spin injection coefficient  $\eta^{(P)}$  with the variation of the Rashba spin-orbit coupling constant (characterized by the Rashba wave number  $k_R \equiv m_s \alpha_R / \hbar^2$  ) in two distinct cases with different bias voltage V. From Figs.2(a)-(c) one can see clearly that the bias voltage may have significant influence on the modulations of the conductance and the magnetoconductance ratio and the spin injection coefficient due to Rashba spin-orbit coupling. From theoretical viewpoints, the spin-transistor effect due to Rashba spin-orbit coupling may depend sensitively on the bias voltage because that the application of a finite bias voltage will not only change the energies of incident electrons ( as in usual electronic devices ) but also have influence on the gate-voltage controlled spin precession in the S channel of the device. The reason for this is that when a finite bias voltage is applied between two terminals of a spin FET, a longitudinal electric field will be established in the semiconductor channel, and due to the presence of this longitudinal electric field, spin precessions of injected spin-polarized electrons in the S channel will depend not only on the gate-voltage controlled Rashba spin-orbit coupling but also depend on the bias voltage. This can be seen clearly from the formulas presented in section II, where we have shown that in the presence of a finite bias voltage, the spinor wave func-

tion in the S region will depend not only on the Rashba spin-orbit coupling constant but also depend on the bias voltage. So, in order to describe correctly the spin precession states of injected spin-polarized electrons in the semiconductor channel of a spin FET, the interplay of the gate-voltage controlled Rashba spin-orbit coupling and the longitudinal electric field induced by the application of a finite bias voltage should be described in a unified way, as was shown in section II. Next, we consider the effect of interfacial scattering. The strength of interfacial scattering can be characterized by two dimensionless parameters defined by  $z_1 = (U_1/\hbar)\sqrt{2m_f/E_F}$  and  $z_2 = (U_2/\hbar)\sqrt{2m_f/E_F}$ , where  $U_1$  and  $U_2$  are the diagonal and off-diagonal elements of the interfacial scattering potential matrix  $\hat{U}$ . The parameters  $z_1$  and  $z_2$  represent the strengths of spin-conserving and spin-flip interfacial scattering, respectively. The effect of interfacial scattering can be seen from Figs3.(a)-(c), where we have plotted the changes of the conductance  $G^{(P)}$  and the spin injection coefficient  $\eta^{(P)}$  and  $\eta^{(AP)}$  with the variation of the bias voltage V in the presence of (spin-conserving and/or spin-flip) interfacial scattering. Fig.3(a) shows that both spin-conserving and spin-flip interfacial scatterings will decrease substantially the conductance of the device, except at some special values of the bias voltage. Figs.3(b)-(c) show that interfacial scattering may decrease the spin injection coefficient if the two ferromagnetic contacts of the device are in parallel configuration and will enhance the spin injection efficiency if the two ferromagnetic contacts of the device are in antiparallel configuration. An interesting fact that can be seen from Figs.3(b)-(c) is that the enhancement of the spin injection efficiency due to interfacial scattering may be very substantial when the two ferromagnetic contacts of the device are in antiparallel configuration, compared with the decrease of the spin injection efficiency due to interfacial scattering when the contacts of the device are in parallel configuration.

In conclusion, in this paper we have discussed the influence of finite bias voltages on the conductance modulations in spin FETs due to Rashba spin-orbit coupling. We have shown that when a finite bias voltage is applied between both terminals of a spin FET, the conductance modulation in the device due to Rashba spin-orbit coupling may depend sensitively on the bias voltage, and in order to describe correctly the spin precession states of injected spin-polarized electrons in the semiconductor channel of the device, the interplay of the gate-voltage controlled Rashba spin-orbit coupling and the bias voltage should be described in a unified way.

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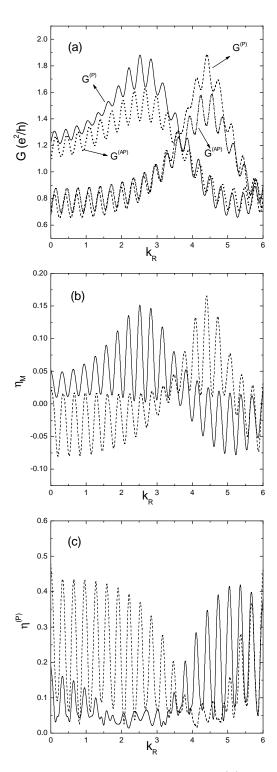


FIG. 2: The changes of (a) the conductance  $G^{(P)}$  and  $G^{(AP)}$ , (b) the magnetoconductance ratio  $\eta_M$ , and (c) the spin injection coefficient  $\eta^{(P)}$ , with the variations of the Rashba wave number  $k_R$  in two cases with different bias voltages. (In Fig2.(a), V=0.1V for the solid and the dotted lines, V=0.2V for the dashed and the dash-dotted lines. In Fig.2(b)-(c), V=0.1V for the solid and V=0.2V for the dashed line. Other parameters used were given in the text. The changes of the spin injection coefficient  $\eta^{(AP)}$  with the variations of  $k_R$  is similar as was shown in Fig.(c) and were not plotted.

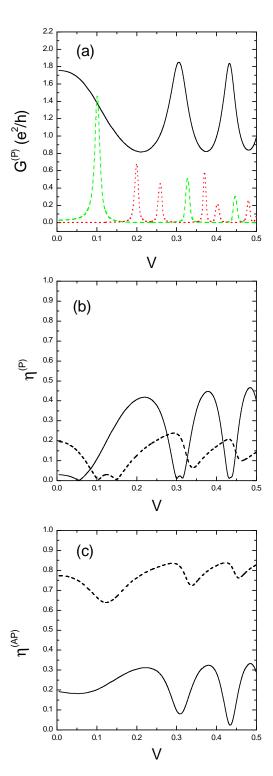


FIG. 3: The changes of the conductance  $G^{(P)}$  and the spin injection coefficient  $\eta^{(P)}$  and  $\eta^{(AP)}$  with the variations of the bias voltage V in the presence of interfacial scattering. (The strength of interfacial scattering are characterized by two dimensionless parameters defined by  $z_1 \equiv (U_1/\hbar)\sqrt{2m_f/E_F}$  and  $z_2 \equiv (U_2/\hbar)\sqrt{2m_f/E_F}$ . In Fig.3(a),  $z_1=0$  and  $z_2=0$  for the solid line;  $z_1=10$  and  $z_1=0$  for the dotted line;  $z_1=0$  and  $z_2=10$  for the dashed line. In Fig3.(b)-(c),  $z_1=0$  and  $z_2=0$  for the solid line;  $z_1=2$  and  $z_2=5$  for the dashed line. Other parameters used were given in the text. The changes of the conductance  $G^{(AP)}$  with the variations of the bias voltage V is similar as was shown in Fig.1(a) and were not plotted.

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